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COMPARATIVE EVALUATION OF COUNT TIME SERIES MODELS (PAR, INAR, DARMA, NB-AR, AND PLN-AR) FOR MODELLING AND FORECASTING WEEKLY CONFIRMED LASSA FEVER CASES IN NIGERIA

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ABSTRACT

Accurate modelling and forecasting of infectious disease counts are essential for effective disease surveillance and public health decision-making. However, epidemic count data often exhibit overdispersion and serial dependence, making conventional count time series models inadequate for reliable prediction. This study comparatively evaluated the performance of five count time series models, namely the Poisson Autoregressive (PAR), Integer-Valued Autoregressive (INAR), Negative Binomial Autoregressive (NB-AR), Poisson Log-Normal Autoregressive (PLN-AR), and Discrete Autoregressive Moving Average (DARMA) models for modelling weekly confirmed Lassa fever cases in Nigeria. Weekly confirmed Lassa fever data comprising 307 observations from January 2020 to December 2025 were obtained from the Nigeria Centre for Disease Control (NCDC). The models were evaluated using simulation experiments and empirical analysis across different partition sizes (20%, 40%, 60%, 80%, and 100%). Model adequacy was assessed using the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC), while forecasting performance was evaluated using the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Continuous Ranked Probability Score (CRPS). Preliminary analyses confirmed that the series was stationary, overdispersed, and exhibited significant temporal dependence. The results showed that the Poisson Log-Normal Autoregressive (PLN-AR) model consistently achieved the lowest information criteria values, indicating superior model fit. The PLN-AR model provided the best overall balance between goodness-of-fit and forecasting performance. The study concludes that the PLN-AR model offers a flexible and robust framework for modelling overdispersed epidemic count data and can substantially improve forecasting accuracy for infectious disease surveillance.

KEYWORDS: Count time series, Lassa fever, Poisson log-normal autoregressive, Negative binomial autoregressive, Integer-valued autoregressive, Discrete autoregressive moving average, Forecasting, Overdispersion.

INTRODUCTION

Infectious diseases remain a major public health challenge worldwide, particularly in developing countries where recurrent outbreaks threaten population health and economic stability. Accurate statistical models are essential for understanding disease transmission dynamics, forecasting outbreaks, and supporting evidence-based public health interventions. Reliable forecasts enable health authorities to strengthen surveillance systems, allocate resources efficiently, and implement timely control measures (World Health Organization [WHO], 2024; Held et al., 2017). Lassa fever is an acute viral haemorrhagic disease endemic to West Africa, with Nigeria accounting for the highest number of reported cases in the region (Richmond & Baglole, 2003; Nigeria Centre for Disease Control [NCDC], 2025). Weekly Lassa fever incidence data are discrete and often exhibit overdispersion and temporal dependence, making conventional time series models less suitable for accurate modelling and forecasting (Fokianos et al., 2009; Cameron & Trivedi, 2013). Consequently, specialized count time series models are required to capture these characteristics. Several count time series models have been developed for analysing dependent count data, including the Poisson Autoregressive (PAR), Integer-Valued Autoregressive (INAR), Negative Binomial Autoregressive (NB-AR), Discrete Autoregressive Moving Average (DARMA), and Poisson Log-Normal Autoregressive (PLN-AR) models (Al-Osh & Alzaid, 1987; Fokianos et al., 2009; Zhu, 2011; Aknouche & Demmouche, 2019). Although these models have been successfully applied to count time series data, previous studies have mainly compared only a few competing models. For instance, Buba et al. (2025) compared the PAR, DARMA, and INAR models and concluded that the PAR model provided superior performance. However, the study did not include the NB-AR and PLN-AR models, which are more suitable for handling overdispersed count data. Consequently, there remains limited empirical evidence comparing the performance of PAR, INAR, DARMA, NB-AR, and PLN-AR within a unified framework using infectious disease count data. This study addresses this gap by comparatively evaluating the five count time series models using Monte Carlo simulation and weekly confirmed Lassa fever incidence data from Nigeria. The models are assessed using information criteria and forecasting accuracy measures to identify the most appropriate model for modelling and forecasting overdispersed Lassa fever count data.

METHODOLOGY

STUDY DESIGN

This study comparatively evaluated the performance of five count time series models for modelling and forecasting weekly confirmed Lassa fever cases in Nigeria. The models considered were the Poisson Autoregressive (PAR), Integer-Valued Autoregressive (INAR), Negative Binomial Autoregressive (NB-AR), Poisson Log-Normal Autoregressive (PLN-AR), and Discrete Autoregressive Moving Average (DARMA) models. The methodological framework consisted of five phases: (i) data collection and preliminary analysis, (ii) model specification and parameter estimation, (iii) simulation study (iv) Empirical Analysis (v) Model evaluation and comparison.

DATA SOURCE AND PRELIMINARY ANALYSIS

Weekly confirmed Lassa fever cases from January 2020 to December 2025 (307 observations) were obtained from the Nigeria Centre for Disease Control (NCDC). Preliminary analyses were conducted to assess the statistical properties of the series, including descriptive statistics, stationarity, overdispersion, temporal dependence, and zero inflation. Stationarity was assessed using the Augmented Dickey–Fuller (ADF) test, while the autocorrelation function (ACF), partial autocorrelation function (PACF), and Box–Ljung test were used to evaluate serial dependence.

MODEL SPECIFICATION

1. Poisson Autoregressive (PAR) Model

The PAR model assumes that the conditional distribution of the response variable follows a Poisson distribution with a conditional mean that depends on previous observations:

The Par (p) Model of order p is defined as $Y_t|\mathcal{F}_{t-1} \sim Poisson(\lambda_t)$ (1)

$P\,(Y_t = y|\mathcal{F}_{t-1}) = \frac{\lambda_t^y e^{-\lambda_t}}{y!}, \quad y = 0,1,2,..$ (2)

Where by $=\ \mathcal{F}_{t-1}$ is the past information data.

2. Poisson Log-Normal Autoregressive (PLN-AR) Model

The PLN-AR model extends the Poisson framework by incorporating a latent log-normal autoregressive process to account for both overdispersion and serial dependence

Observationeqn.$Y_t \left| \lambda_t \sim Poisson \left(\lambda_t\right),\ t = 1\,,2,\right.$ ....(3)

Latent Process (Log – Normal AR (1); $\log\,(\lambda_t) = \mu + \phi\log(\lambda_{t-1}) + \varepsilon_t$ (4)

Where λ_t; intensity (mean of poisson)

μ ; intercept, ϕ; Autoregressive parameter ($|\phi| < 1$) $\varepsilon_t \sim N(0,\sigma 2)$; Gaussian noise

3. Integer-Valued Autoregressive (INAR) Model

The INAR model uses the binomial thinning operator to model dependence in integer-valued time series.

Defined as: $Y_t = \alpha\ o\ Y_{t-1} + \varepsilon_t$, $0 \leq \alpha \leq 1$ (5)

Where: o is the binomial thinning operator, $B_{t,i} \sim Bernoulli\ (\alpha),\ i.i.d.\ \varepsilon_t$ is the innovation process, $\alpha \in (0,1)$: autoregressive parameter.

4. Negative Binomial Autoregressive NB-AR Model

The NB-AR model assumes a negative binomial conditional distribution, allowing the variance to exceed the mean through an additional dispersion parameter.

$Y_t|\mathcal{F}_{t-1} \sim Negative\ Binomial\ (\mu_t, k),$

(6)

Where: $(\mathcal{F}_{t-1}, Y_{t-2,...})$ is the information set.$\mu_t = E(Y_t|\mathcal{F}_{t-1})$, k

$\geq C\ is\ the\ dispersion\ (size)\ \ parameter,$ $0 < \mu_t < 1$ is the success probability.

5. Discrete Autoregressive Moving Average (DARMA) **Model**

The DARMA model combines autoregressive and moving-average components for modelling discrete-valued time series.

$$X_t = \sum_{i=1}^{p} \alpha_i \ o \ X_{t-1} + \sum_{j=1}^{q} B_j \ o \ \varepsilon_{t-j} + \varepsilon_t \quad (7)$$

Where: $X_t = Observed\ count\ process.\ \alpha_i \epsilon [0,1] = Autoregressive\ parameters.$

$B_j\ \epsilon[0,1] = Moving\ Average\ parameters.\ \varepsilon_t = Innovation\ (i.i.d).\ o = B.Thinning$

SIMULATION STUDY

A simulation study was conducted to compare the performance of the five count time series models under controlled conditions before their application to the real Lassa fever data. Synthetic count time series data exhibiting overdispersion and temporal dependence were generated using Monte Carlo simulation. The competing models were evaluated using AIC, BIC, HQIC, RMSE, MAE, MAPE, and CRPS across different data partitions (20%, 40%, 60%, 80%, and 100%) to identify the model with the best fit and forecasting performance.

Table. 1.

Model	AIC	BIC	HQIC
PLN-AR	1886.0461	1896.5621	1890.2804
NB-AR	1949.3953	1959.8990	1953.6251
INAR	3749.1030	3756.1140	3751.9260
PAR	4662.6530	4669.6630	4665.4760
DARMA	5158.8380	5166.2910	5161.8180

Interpretation: The PLN-AR model achieved the lowest AIC (1886.05), BIC (1896.56), and HQIC (1890.28), indicating the best overall fit among the five competing models. The NB-AR model ranked second, while the INAR, PAR, and DARMA models recorded higher information criterion values, reflecting comparatively poorer model performance.

SIMULATED COMPARATIVE MATRIX OF PLN-AR AT DIFFERENT PARTITIONED LEVEL

The best fitted model was test at different sample size (20%, 40%, 60%, 80%, and 100%) partitioned of the generated data;

Table. 2

Model	Partition	TotalN	TrainN	AIC	BIC	HQIC	RMSE	MAE	CRPS
PLN-AR	20%	61	49	437.8811	443.5565	440.0343	8.5172	7.0378	5.2294
PLN-AR	40%	123	98	795.5995	803.3544	798.7362	56.8081	48.1615	36.2336
PLN-AR	60%	184	147	1265.9028	1274.8741	1269.5479	13.0027	11.8567	8.5044
PLN-AR	80%	246	197	1593.9719	1603.8215	1597.9591	10.4194	9.7192	7.6488
PLN-AR	100%	307	246	1886.0461	1896.5621	1890.2804	8.2217	7.4770	5.7690

Interpretation; Interpretation: Table 2 shows that the forecasting performance of the PLN-AR model generally improved with increasing sample size. The 100% partition achieved the lowest RMSE (8.22), MAE (7.48), and CRPS (5.77), indicating the highest forecasting accuracy, while the 40% partition recorded the largest forecast errors. Overall, the PLN-AR model produced more reliable forecasts with larger datasets.

SIMULATION FORECAST OF PLN-AR MODEL SELECTED AT 100%

The simulated data generated were shown on the best model forecasting accuracy. The plot below demonstrates the forecasting accuracy of the PL-AR model chooses at 100%;

Fig. 2. PLN-AR Model At 100% Simulation Forecast.

The plot compares the actual simulated test data with the PLN-AR forecasts. The model captures the overall trend and most low to moderate counts but fails to accurately predict some high peaks. The RMSE (7.615) and MAE (5.164) indicate relatively small prediction errors, while the high MAPE (133.89%) is mainly due to the presence of many low count values. Overall, the PLN-AR model provides a reasonable forecast but struggles with extreme values.

EMPIRICAL ANALYSIS

Weekly Lassa fever Time Series Plot of the Data

The time series plot shows the weekly reported cases of Lassa fever over a period of approximately six years with the total observations:307

Fig.1. Weekly Lassa Fever Cases Plot Interpretation: The weekly Lassa fever cases exhibit a clear seasonal pattern with recurring annual outbreak peaks and rapid declines, indicating strong temporal dependence and suitability for time series modelling and forecasting.

DISPERSION AND DIAGNOSTIC CHECKS

The data are highly overdispersed (dispersion ratio = 32.65), with negligible zero inflation (0.65%). The ADF test confirmed stationarity, while the ACF/PACF and Box–Ljung tests indicated significant AR (1) temporal dependence, making the series suitable for count time series modelling and forecasting.

COMPARISON ANALYSIS OF FIVE MODELS BY LASSA FEVER CONFIRMED CASES

The Lassa fever confirmed cases were analyzed using the five count time series model to asses the best model fit and forecasting accuracy of the selected model to be determine. The below table compares the evaluation of five competing models chosen at 100% sample size to give out the reliable and generous framework analysis;

Table. 3

Model	AIC	BIC	HQIC	RMSE	MAE	MAPE (%)	CRPS
NB-AR	1648.1195	1658.6355	1648.9424	21.1091	15.0955	106.8519	11.0430
PLN-AR	1619.1519	1629.6679	1623.3862	23.7111	13.1481	53.4460	12.2153
PAR	2161.9674	2168.9781	2164.7903	21.1091	15.0955	106.8519	12.9266
INAR	2161.9674	2168.9781	2164.7903	21.1091	15.0955	106.8519	12.9266
DARMA	2756.5090	2763.9625	2759.4895	11.6196	7.8409	89.0530	6.1036

The results indicate that the PLN-AR model provided the best model fit based on AIC, BIC, and HQIC, the forecasting accuracy for PLN-AR should be considered. The NB-AR model ranked second in model fit and exhibited competitive forecasting performance, outperforming the PAR and INAR models. Consequently, the introduced models (NB-AR and PLN-AR) generally performed better than the existing PAR, INAR and DARMA models, while PLN-AR emerged as the preferred model for fitting and forecasting the Lassa fever count data.

TRADITIONAL MODEL (PAR) PERFORMANCE WITH THE NEWLY INTRODUCED MODELS (NB-AR AND PLN-AR)

The best existing model (PAR) were compared with newly introduced model (NB-AR and PLN-AR) across different partition sizes (20%, 40%, 60%, 80%, and 100%) were assessed. The table below presents a comparative summary of the model fit and forecasting performance based on key evaluation criteria, including AIC, BIC, HQIC, RMSE, MAE, MAPE, and CRPS.

Table. 4.

Partition	Model	AIC	BIC	HQIC	RMSE	MAE	MAPE (%)	CRPS
20%	NB-AR	335.7908	341.4662	335.2263	6.1236	3.9009	23.9306	3.3919
	PLN-AR	322.0969	327.7724	324.2502	10.2016	7.5511	44.1660	6.4934
	PAR	433.3153	437.0990	434.7508	6.1236	3.9009	23.9306	2.9462
40%	NB-AR	629.7512	637.5061	629.8423	32.7746	23.1329	80.8094	19.8110
	PLN-AR	614.7353	622.4902	617.8720	36.8532	26.5073	72.5188	25.1037
	PAR	794.2776	799.4476	796.3687	32.7746	23.1329	80.8094	21.3374
60%	NB-AR	953.5583	962.5296	953.9884	36.1455	21.8632	157.4407	18.5132
	PLN-AR	933.7717	942.7430	937.4169	39.7012	23.3911	99.9787	22.1239
	PAR	1189.4661	1195.4469	1191.8961	36.1455	21.8632	157.4407	19.9596
80%	NB-AR	1307.9614	1317.8110	1308.6196	28.1746	18.4069	125.7034	14.5387
	PLN-AR	1285.4101	1295.2597	1289.3973	31.1396	17.8216	72.6106	16.5515
	PAR	1717.7977	1724.3641	1720.4559	28.1746	18.4069	125.7034	16.3757
100%	NB-AR	1648.1195	1658.6355	1648.9424	21.1091	15.0955	106.8519	11.0430
	PLN-AR	1619.1519	1629.6679	1623.3862	23.7111	13.1481	53.4460	12.2153
	PAR	2161.9674	2168.9781	2164.7903	21.1091	15.0955	106.8519	12.9266

Interpretation: Across all partition sizes, the PLN-AR model consistently achieved the lowest AIC, BIC, and HQIC values, indicating superior model fit. However, the PLN-AR model forecasting performance were considered across the competing model forecasting. The PAR model showed substantially larger information criteria values than both NB-AR and PLN-AR, indicating a poorer fit to the data. Overall, the results suggest that PLN-AR is the preferred model for fitting and forecasting the Lassa fever count data.

FORECAST PLOT OF LASSA FEVER CASES BY PLN-AR MODEL AT 100%

Fig. 2. PLN-AR Model Forecast at 100%: The PLN-AR model produced smoother forecasts and achieved the lowest MAE and MAPE, indicating the best point forecasting accuracy, although it still underestimated the epidemic peak.

SUMMARY OF FINDINGS

This study compared the PAR, INAR, NB-AR, PLN-AR, and DARMA models for modelling and forecasting weekly confirmed Lassa fever cases in Nigeria. The results showed that the data were overdispersed and temporally dependent, making flexible count time series models more appropriate. Among the competing models, the PLN-AR model consistently achieved the best model fit and forecasting performance, while the NB-AR model ranked second. The PAR, INAR, and DARMA models exhibited comparatively poorer performance.

CONCLUSION

The findings indicate that the PLN-AR model is the most suitable model for modelling and forecasting weekly Lassa fever incidence in Nigeria because it effectively captures overdispersion and temporal dependence. Its superior performance suggests that it can provide more reliable forecasts to support disease surveillance, epidemic preparedness, and evidence-based public health decision-making.

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