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**PERFORMANCE EVALUATION OF MULTILEVEL LDPC-CODED  
OFDM OVER WIRELESS FADING CHANNELS**

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**\*Aziz Yilmaz****Article Received: 18 June 2026, Article Revised: 08 July 2026, Published on: 28 July 2026****\*Corresponding Author: Aziz Yilmaz**Doi: <https://doi-doi.org/101555/ijarp.2098>**ABSTRACT**

Orthogonal Frequency Division Multiplexing (OFDM) is one of the commonly used transmission techniques in today's wireless communication systems due to its high spectral efficiency, resilience to multipath propagation conditions, and support for high data transmission rates. However, the additive white Gaussian noise (AWGN) and Rayleigh and Rician fading that occur in wireless communication channels can significantly negatively impact the bit error rate (BER) performance of OFDM systems. Therefore, the use of advanced channel coding techniques aimed at enhancing transmission reliability is of great importance. Low-Density Parity-Check (LDPC) codes are one of the prominent channel coding methods in modern communication systems due to their near-Shannon limit error correction performance and iterative decoding algorithms. In this study, the performance of an OFDM system using multi-level LDPC coding under different wireless channel conditions has been examined in a simulation environment. The developed system model consists of an LDPC encoder, an OFDM transmitter, a wireless channel model, an OFDM receiver, and an iterative LDPC decoder. The performance of the system was evaluated using the Bit Error Rate (BER) criterion for different signal-to-noise ratio (SNR) values under AWGN, Rayleigh, and Rician channel models. The obtained simulation results show that multi-level LDPC coding significantly improves error performance under all channel conditions compared to the uncoded OFDM system. While the highest performance was achieved in the AWGN channel, it was also observed that multi-level LDPC coding significantly reduced the error effects caused by multipath propagation in Rayleigh and Rician fading channels. The results indicate that the proposed approach offers an effective channel coding solution for enhancing the reliability of OFDM-based wireless communication systems. This study aims to contribute to the design of OFDM-based communication systems by holistically evaluating the performance of multi-level LDPC coding under different wireless channel conditions and

providing a reference that could serve as a foundation for future advanced channel coding studies.

**KEYWORDS:** OFDM, Low-Density Parity-Check Codes (LDPC), Multi-Level LDPC, Wireless Communication, Bit Error Rate, AWGN, Rayleigh Fading, Rician Fading.

## 1. INTRODUCTION

The increasing data rate requirements, high bandwidth needs, and expectations for reliable communication in wireless communication systems have made the development of transmission techniques with high spectral efficiency necessary. Orthogonal Frequency Division Multiplexing (OFDM) is widely used today in digital television broadcasting, broadband wireless access systems, Wi-Fi, LTE, and other multi-carrier communication applications due to its high data transmission capacity, resilience to frequency-selective fading, and ability to mitigate the adverse effects of multipath propagation. However, OFDM systems require strong error correction techniques due to the increase in bit error rate, especially under noisy and fading wireless channel conditions.

Channel coding methods play a fundamental role in enhancing transmission reliability. In recent years, Low-Density Parity-Check (LDPC) codes have been considered one of the most successful channel coding techniques due to their error correction performance close to Shannon capacity and iterative decoding algorithms. Especially in communication systems with high data rates, the error correction capability provided by LDPC codes significantly contributes to increasing system reliability.

The combined use of OFDM and LDPC coding allows for lower error rates in wireless environments dominated by multipath propagation. However, the systematic evaluation of the performance of multi-level LDPC coding under different channel conditions remains important for communication system designers. Especially, the demonstration of performance differences obtained under AWGN, Rayleigh, and Rician channel models contributes to the evaluation of the effectiveness of channel coding methods.

In this study, the error performance of an OFDM communication system using multi-level LDPC coding under different wireless channel models was examined using simulation methods. The developed system model was implemented in the MATLAB environment; Bit Error Rate (BER) performance was analysed under different signal-to-noise ratios for AWGN, Rayleigh, and Rician channel conditions. The obtained results were evaluated

comparatively, and the effectiveness of multi-level LDPC coding in enhancing the reliability of OFDM-based wireless communication systems was demonstrated.

The main contributions of this study can be summarised as follows:

- An integrated simulation model of the multi-level LDPC coded OFDM system has been created.
- The system performance has been comparatively evaluated under AWGN, Rayleigh, and Rician channel models.
- The impact of channel conditions on system performance was demonstrated by analysing the BER results obtained at different SNR levels.
- The effectiveness of multi-level LDPC coding in improving error performance in wireless communication systems has been experimentally demonstrated.

## 2. LITERATURE

The theoretical foundation of reliable data transmission in wireless communication systems is based on Shannon's channel capacity approach, which demonstrates that communication can be established over a noisy channel with the probability of error reduced as desired (Shannon, 1948). This approach has made the development of error correction codes that provide performance close to channel capacity one of the fundamental research areas of communication theory. Linear block codes, convolutional codes, and modern graph-based codes have been developed in this direction, offering different balances between coding gain, code rate, decoding complexity, and delay (Lin & Costello, 2004; Moreira & Farrell, 2006; Proakis & Salehi, 2008; Ryan & Lin, 2009).

Low-Density Parity-Check codes were first defined by Gallager (1962) as linear block codes based on sparse parity-check matrices. The structural representation of LDPC codes was later addressed by Tanner (1981) through bipartite graphs, and the relationships between variable nodes and check nodes were systematically explained. The Tanner graph structure allows the decoding process to be carried out in the form of iterative message exchange between nodes. Kschischang et al. (2001) detailed the probabilistic foundation of LDPC decoding processes by presenting the general framework of the sum-product algorithm in graphical models. These studies have established the theoretical foundation for the application of LDPC codes using low-complexity iterative decoding algorithms.

LDPC codes found limited application in the early period when they were first proposed due to their high memory and processing requirements. With the advancements in computer technologies, MacKay and Neal (1996) have brought the high error correction capability of

sparse graph codes back into discussion. MacKay (1999) demonstrated that well-designed LDPC codes can perform close to the Shannon limit. Richardson and Urbanke (2001) examined the capacity of iterative decoding using the density evolution method; Richardson et al. (2001) demonstrated the decisive impact of degree distributions on the performance in the design of irregular LDPC codes. Chung et al. (2001) reported that with optimized irregular LDPC codes, error performance very close to the AWGN channel capacity could be achieved. These findings have accelerated the transition from regular LDPC codes to irregular and channel condition-optimized code structures.

At finite block lengths, the performance of LDPC is dependent not only on the code rate but also on the cycle structure of the parity-check matrix and the node connections. Especially short cycles can negatively affect convergence performance by disrupting the independence of messages during iterations. Fossorier (2004) examined the quasi-cyclic LDPC codes formed from circulant permutation matrices and revealed the balance between the ease of application and performance of structured codes. Richardson et al. (2001) and Richardson and Urbanke (2001) demonstrated the relationship between degree distribution and decoding threshold; these studies highlighted the need to jointly consider threshold analysis, error floor, and decoding complexity in LDPC code design.

The multi-level coding approach is based on the principle of dividing high-degree signal sets into bit layers with different levels of protection. Imai and Hirakawa (1977) defined the fundamental structure of multi-level coding and multi-stage decoding and demonstrated that different codes could be used at each bit level. Ungerboeck (1982) demonstrated that coding gain could be achieved without increasing bandwidth by using the set partitioning approach in multi-level signal constellations. Zehavi (1992) demonstrated the diversity advantage of bit-interleaved coded modulation in fading channels; Caire et al. (1998) comprehensively examined the capacity and error probability characteristics of bit-interleaved coded modulation. These studies have laid the foundation for the application of LDPC codes to different modulation levels and the varying protection of bit levels according to channel conditions.

The fundamentals of the OFDM technique are based on parallel data transmission over a large number of orthogonal subcarriers. Weinstein and Ebert (1971) demonstrated that the discrete Fourier transform could be used efficiently in multicarrier data transmission. Bingham (1990) examined the advantages of multicarrier modulation in frequency-selective channels and its practical application conditions. Van Nee and Prasad (2000) examined the dimensions of system design in wireless multiple access communication using OFDM in

detail, while Wang and Giannakis (2000) explained the relationship between multicarrier communication, Fourier analysis, and information theory. The main advantage of OFDM is that it divides a frequency-selective channel into a large number of narrowband subchannels that are approximately flat fading. The use of a cyclic prefix reduces inter-symbol interference and facilitates receiver equalisation; however, noise, channel estimation errors, and deep fades can significantly limit BER performance (Goldsmith, 2005; Proakis & Salehi, 2008).

The use of channel coding in OFDM systems contributes to the correction of errors in subcarriers affected by deep fading during the decoding stage. Futaki and Ohtsuki (2001) conducted one of the pioneering studies demonstrating that the LDPC-coded OFDM structure improves error performance in frequency-selective fading channels. The researchers then examined LDPC-OFDM systems using M-PSK modulation to evaluate the impact of signal mapping and channel conditions on BER (Futaki & Ohtsuki, 2002). The aforementioned studies have shown that LDPC coding is an effective forward error correction method in multi-carrier systems.

The performance of coded OFDM systems is influenced by the code structure, as well as channel correlation, spreading method, modulation degree, and the accuracy of channel state information. Snow et al. (2007) developed error rate analysis methods for coded multi-carrier systems over frequency-selective quasi-static fading channels, demonstrating that the correlation between subcarriers should be considered in BER predictions. Serener and Nassar (2007) examined the performance of short block length LDPC codes in OFDM and spread OFDM structures under frequency-correlated Rayleigh channels. Stiglmayr et al. (2008) evaluated the approach of adaptive coding and modulation based on mutual information in bit-interleaved OFDM systems using punctured LDPC codes. These studies show that systems adaptable to channel conditions, instead of fixed coding and modulation structures, can create a more suitable balance between bandwidth efficiency and error performance.

The performance of traditional LDPC structures in fading channels can vary significantly due to the exposure of coded bits to different channel conditions. Boutros et al. (2010) demonstrated that root-check LDPC codes, which provide full diversity for non-ergodic block fading channels, can achieve performance close to the cut-off limit by developing an appropriate graph structure. This result indicates that optimizing only for the AWGN channel in LDPC design is not sufficient; the channel's fading structure must also be reflected in the parity-check matrix. However, in practice, the integration of standard or traditional LDPC codes with OFDM remains important due to design simplicity and low decoding complexity.

When the current literature is generally evaluated, a significant portion of the studies focuses on single-level LDPC-OFDM structures, specific modulation schemes, or a single channel model. Comparative studies in which the multi-level LDPC coding structure is examined together under AWGN, Rayleigh, and different Rician channel conditions are more limited. Additionally, examining the impact of different generator matrices used in two and three-level coding structures on error performance within the same simulation framework holds particular significance for system design. Therefore, the present study aims to provide an application-oriented contribution to the literature by comparatively evaluating the BER behaviour of a multi-level LDPC coded OFDM system under different noise and fading conditions.

### 3. METHODOLOGY

#### 3.1 Research Approach

In this study, the performance of multi-level Low-Density Parity-Check (LDPC) coding in OFDM-based wireless communication systems has been examined using simulation methods. The research is a quantitative and comparative simulation study aimed at revealing the impact of multi-level LDPC coding on the system's error performance under different wireless channel conditions.

The system model developed in the study consists of random bit generation, multi-level LDPC coding, digital modulation, OFDM signal generation, wireless channel modeling, and iterative LDPC decoding performed on the receiver side. In performance evaluations, the primary success metric used was the Bit Error Rate (BER).

#### 3.2 Generation of the OFDM Signal

The OFDM system transmits high-speed data streams by distributing them over a large number of mutually orthogonal subcarriers. Data symbols created in the frequency domain are converted to the time domain using the Inverse Fast Fourier Transform (IFFT).

An OFDM symbol is expressed as follows.

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}$$

Where;

- N is the number of subcarriers,
- X(k) represents the data symbols in the frequency domain,

- $x(n)$  represents the OFDM signal in the time domain.

To reduce inter-symbol interference caused by multipath propagation, a cyclic prefix has been added to each OFDM symbol.

### 3.3 Multi-Level LDPC Coding

In the study, a multi-level LDPC coding approach was used to enhance error correction performance. LDPC codes are linear block codes with low-density parity-check matrices and offer error performance very close to the Shannon limit thanks to iterative decoding algorithms.

The encoding process will be,

$$c=UG$$

Where;

- $u$  is the information bit vector,
- $G$  is the generator matrix,
- $C$  is the encoded bit sequence.

On the receiver side, the information bits are retrieved using a probability-based iterative decoding algorithm.

### 3.4 Wireless Channel Models

The proposed system has been tested under three different channel models.

#### 3.4.1 AWGN Channel

The AWGN model is the basic channel model that considers only additive white Gaussian noise.

Received signal will be,

$$r(t)=s(t)+n(t)$$

Where;

- $s(t)$  Transmitted Signal,
- $n(t)$  Gauss Noise,
- $r(t)$  Received Signal.

#### 3.4.2 Rayleigh Channel

The Rayleigh channel model represents multipath propagation environments where there is no direct line of sight.

Received signal will be,

$$r(t)=h(t).s(t)+n(t)$$



Here,  $h(t)$  represents the channel coefficient with a Rayleigh distribution.

### 3.4.3 Rician Channel

The Rician channel model represents wireless communication environments where a direct line of sight is present. In this model, the received signal consists of both the direct and reflected signal components.

### 3.5 Performance Metric

The system's performance has been evaluated using the Bit Error Rate (BER). BER is calculated using the following equation.

$$BER = \frac{N_{error}}{N_{total}}$$

Here;

- $N_{error}$  represents the number of bits incorrectly decoded,
- $N_{total}$  represents the total number of bits sent.

By comparing the BER results obtained at various Signal-to-Noise Ratio (SNR) values for different channel models, the contribution of multi-level LDPC coding to system performance has been analysed.

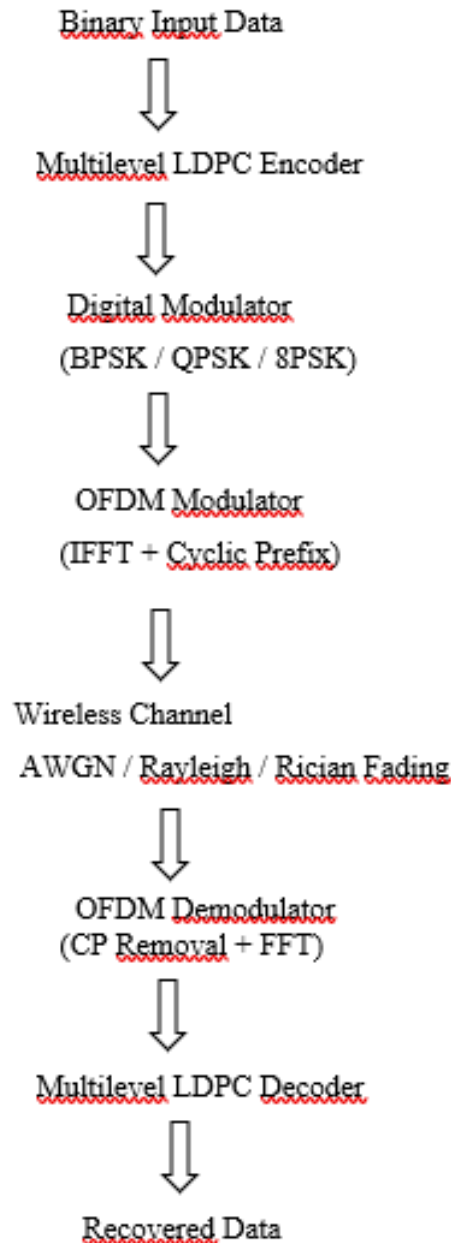
### 3.6 Simulation Environment

The proposed system was implemented in the MATLAB environment, and all simulations were conducted using the same system parameters. In the comparative analyses, only the channel model was changed, while all other parameters were kept constant. Thus, the performance differences obtained were reliably evaluated to be solely due to the channel conditions and the multi-level LDPC coding structure.

## 4. FINDINGS AND DISCUSSION

The performance of the proposed multi-level LDPC coded OFDM system in this study was evaluated through simulation under AWGN, Rayleigh, and Rician fading wireless channel models. The system performance was analysed using the Bit Error Rate (BER) results obtained at different Signal-to-Noise Ratio (SNR) values. The obtained results reveal the effects of both channel conditions and the multi-level LDPC coding structure on system performance.

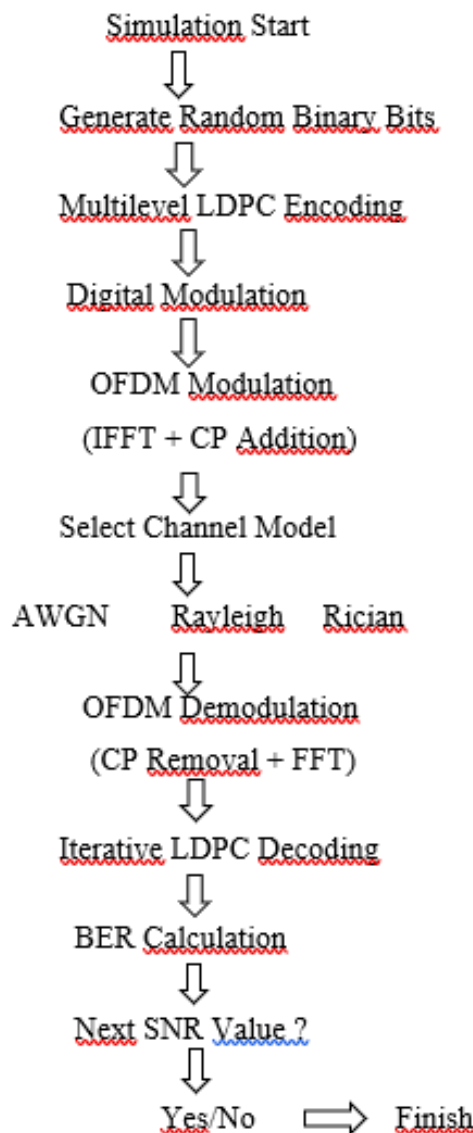




**Figure 1. Proposed Multilevel LDPC Coded OFDM Communication System.**

Figure 1 shows the overall architecture of the communication system used in the study. The system begins with the binary data sequence, which is randomly generated, being passed through a multi-level LDPC encoder. The encoded bits are converted into symbols using the selected digital modulation method, and OFDM signals are generated with the help of the IFFT process. After the cyclic prefix is added, the signals are transmitted through AWGN, Rayleigh, or Rician channel models. On the receiver side, after the cyclic prefix is removed, the FFT process is applied, and the information bits are retrieved using an iterative LDPC

decoder. This structure has been used to evaluate the error performance of multi-level LDPC coding under different channel conditions.



**Figure 2. Simulation Flow Diagram of the Multi-Level LDPC Coded OFDM System.**

The flow of the simulation process carried out in Figure 2 is shown. The simulation begins with the generation of random binary data sequences. Then, multi-level LDPC coding is applied, digital modulation is performed, and OFDM symbols are generated. After the signals are transmitted over the selected channel model, OFDM decoding and iterative LDPC decoding processes are carried out on the receiver side. In the final stage, the sent and received bit sequences are compared to calculate the Bit Error Rate (BER). This process is repeated for different SNR values to obtain system performance curves.

#### 4.1 Performance in the AWGN Channel

In the first stage, the system was tested under the AWGN channel model, which only contains additive white Gaussian noise. The AWGN channel represents an ideal communication environment where multipath fading effects are absent, thus allowing for the evaluation of the basic performance of the proposed system.

Simulation results show that the BER values decrease regularly with the increase in SNR values. Especially in the medium and high SNR regions, it has been observed that the error correction performance of multi-level LDPC coding significantly increases. This situation is due to the iterative LDPC decoding algorithm being able to work with more reliable probability information.

The results obtained in the AWGN channel show that multi-level LDPC coding improves the basic error performance of the OFDM system and that the gain provided by channel coding becomes more pronounced under ideal channel conditions.

#### 4.2 Performance in the Rayleigh Channel

The Rayleigh channel model represents wireless communication environments where there is no direct line of sight and multipath propagation is dominant. Therefore, the system performance has been examined under more challenging conditions compared to the AWGN channel.

Simulation results show that the BER values in the Rayleigh channel are higher compared to the AWGN channel due to multipath fading. Especially in low SNR regions, sudden changes in channel gain increase the probability of errors. However, significant improvements in system performance have been achieved thanks to the iterative error correction capability of multi-level LDPC coding.

With the increase in SNR value, a rapid improvement in the BER curves was observed, thus it was seen that multi-level LDPC coding significantly reduced the disruptive effects caused by the channel.

#### 4.3 Performance in the Rician Channel

The Rician channel model represents wireless communication environments where a direct line of sight is present and multipath components are simultaneously effective. This channel structure offers more favourable transmission conditions compared to the Rayleigh channel.

The obtained results show that the BER performance in the Rician channel lies between the AWGN and Rayleigh channels. The presence of a direct line of sight has contributed to the reduction of fluctuations in the signal level, thereby lowering the error rates.

Multilevel LDPC coding has also improved error correction performance in the Rician channel and has made significant contributions to reliable data transmission, especially in the medium to high SNR regions.

#### **4.4 Comparison of Channel Models**

When the three different channel models are evaluated together, it is observed that the system performance varies depending on the channel characteristics.

Since the AWGN channel only has the effect of noise, the lowest BER values were obtained. The Rayleigh channel, on the other hand, created the most challenging channel conditions due to multipath fading and caused the highest error rates. The Rician channel, with the contribution of the direct line of sight, has exhibited a more successful performance compared to the Rayleigh channel.

These results show that multi-level LDPC coding increases system reliability not only under ideal channel conditions but also in fading wireless environments.

#### **4.5 Evaluation of the Impact of Multi-Level LDPC Coding**

The multi-level LDPC structure used in the study contributes to reducing the probability of errors occurring during transmission by allowing the processing of information bits through multiple coding layers. Especially thanks to the probability updates provided by the iterative solving algorithm, the error correction performance is significantly improved.

When evaluating the simulation results, it is observed that multi-level LDPC coding consistently improves system performance under different channel conditions. The more pronounced decrease in error rates with the increase in SNR values indicates that the coding approach used provides a suitable solution for OFDM-based communication systems that require high reliability.

#### **4.6 DISCUSSION**

The findings obtained indicate that multi-level LDPC coding is an effective channel coding method that improves error performance in OFDM-based communication systems. Especially under fading wireless channel conditions, the results obtained show that the iterative LDPC decoding algorithm significantly contributes to error correction performance.

The results of the study are generally consistent with previous works that demonstrate the use of strong channel coding techniques in multi-carrier communication systems increases data transmission reliability. However, the joint evaluation of the multi-level LDPC structure under different channel models in this study has allowed for a comprehensive analysis of the system behaviour.

In conclusion, the proposed structure can be considered an effective channel coding approach that can be used in wireless communication applications where the error rate is critical.

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